

RENORMALIZABILITY OF A MODIFIED GENERALLY COVARIANT YANG-MILLS ACTION

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ABSTRACT

A modified generally covariant Yang-Mills action, which depends on the complex structure of spacetime and not its metric, is proved to be renormalizable. This proof makes this Lagrangian model the unique known generally covariant four dimensional model to be renormalizable without higher order derivatives. The first order one-loop diagrams are computed in an appropriate gauge condition and they are found to be finite.

1 INTRODUCTION

Renormalizability seems to be the necessary criterion for a Quantum Field Theoretic Lagrangian to be self consistent. Any physically interesting Lagrangian has to be renormalizable in order to provide finite computations of physical quantities. Recall that one of the cornerstones of the success of the Standard Model was the proof of its renormalizability. But the straightforward “covariantization” of the Standard Model action with the Einstein gravitational term is not renormalizable. Therefore, this route has been abandoned as a possible unification of Gravity and Quantum Field Theory. It is well known that the main argument of the superstrings researchers is that superstrings bypass renormalizability problem. In the present work a slightly modified generally covariant Yang Mills action is found to be renormalizable.

The covariantized ordinary Yang Mills action

$$I_{YM} = -\frac{1}{4} \int d^4x \sqrt{-g} g^{\mu\nu} g^{\rho\sigma} F_{j\mu\rho} F_{j\nu\sigma} \quad (1.1)$$

$$F_{j\mu\nu} = \partial_\mu A_{j\nu} - \partial_\nu A_{j\mu} - q f_{jik} A_{i\mu} A_{k\nu}$$

is invariant under the Weyl transformation, but it is not renormalizable, because the regularization procedure generates the conformally invariant geometric term

$$I_W = \int d^4x \sqrt{-g} C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma} \quad (1.2)$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor. The quantization of this action leads to inconsistencies, because its explicit dependence on second order derivatives generates negative norm states. Despite the failure to provide a self-consistent Quantum Field Theory we see that the Weyl symmetry restricts all the permitted geometric action terms to just one, the (1.2). The renormalizability of the Lagrangian model considered in this work is essentially based on an extended Weyl symmetry over the null tetrad, which does not permit even this geometric action (1.2).

The initial idea was an effort[7] to find a four-dimensional action which depends on the complex structure and not on the metric of the spacetime. Recall that the two-dimensional string action has exactly this property. Its form

$$I_S = \frac{1}{2} \int d^2\xi \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \quad (1.3)$$

does not essentially depend on the metric $\gamma^{\alpha\beta}$ of the 2-dimensional surface, but it depends on its structure coordinates (z^0, \bar{z}^0) , because in these coordinates it takes the form

$$I_S = \int d^2z \partial_0 X^\mu \bar{\partial}_0 X^\nu \eta_{\mu\nu} \quad (1.4)$$

All the wonderful properties of the string model are essentially based on this characteristic feature of the string action.

In the case of a four dimensional manifold the structure coordinates are $(z^\alpha, z^{\tilde{\alpha}})$, $\alpha = 0, 1$ and the complex structure preserving transformations are $z'^\alpha = z'^\alpha(z^\beta)$, $z'^{\tilde{\alpha}} = z'^{\tilde{\alpha}}(z^{\tilde{\beta}})$. The invariant action of the model is

$$I_G = \int d^4z F_{j01} F_{j\tilde{0}\tilde{1}} + \text{comp. conj.} \quad (1.5)$$

$$F_{jab} = \partial_a A_{jb} - \partial_a A_{jb} - q f_{jik} A_{ia} A_{kb}$$

Using the null tetrad one can transcribe[8] this action to a the following generally covariant form

$$I_G = \int d^4x \sqrt{-g} \{ (\ell^\mu m^\rho F_{j\mu\rho}) (n^\nu \bar{m}^\sigma F_{j\nu\sigma}) + (\ell^\mu \bar{m}^\rho F_{j\mu\rho}) (n^\nu m^\sigma F_{j\nu\sigma}) \} \quad (1.6)$$

$$F_{j\mu\nu} = \partial_\mu A_{j\nu} - \partial_\nu A_{j\mu} - q f_{jik} A_{i\mu} A_{k\nu}$$

where $A_{j\mu}$ is a gauge field and $(\ell_\mu, n_\mu, m_\mu, \bar{m}_\mu)$ is an integrable null tetrad. The difference between the present action and the ordinary Yang-Mills action becomes more clear in the following form of the action.

$$I_G = -\frac{1}{8} \int d^4x \sqrt{-g} (2g^{\mu\nu} g^{\rho\sigma} - J^{\mu\nu} J^{\rho\sigma} - \bar{J}^{\mu\nu} \bar{J}^{\rho\sigma}) F_{j\mu\rho} F_{j\nu\sigma} \quad (1.7)$$

where $g_{\mu\nu}$ is the metric and J_μ^ν is the tensor of the integrable complex structure derived from the null tetrad[2] using the following relations

$$g_{\mu\nu} = \ell_\mu n_\nu + n_\mu \ell_\nu - m_\mu \bar{m}_\nu - \bar{m}_\mu m_\nu \quad (1.8)$$

$$J_\mu^\nu = i(\ell_\mu n^\nu - n_\mu \ell^\nu - m_\mu \bar{m}^\nu + \bar{m}_\mu m^\nu)$$

The integrability condition of the complex structure implies the Frobenius integrability conditions of the pairs (ℓ_μ, m_μ) and (n_μ, \bar{m}_μ) . That is

$$(\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu \ell_\nu) = 0 \quad , \quad (\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu m_\nu) = 0 \quad (1.9)$$

$$(n^\mu m^\nu - n^\nu m^\mu)(\partial_\mu n_\nu) = 0 \quad , \quad (n^\mu m^\nu - n^\nu m^\mu)(\partial_\mu m_\nu) = 0$$

Frobenius theorem states that there are four complex functions $(z^\alpha, z^{\tilde{\alpha}})$, $\alpha = 0, 1$, such that

$$dz^\alpha = f_\alpha \ell_\mu dx^\mu + h_\alpha m_\mu dx^\mu \quad , \quad dz^{\tilde{\alpha}} = \tilde{f}_{\tilde{\alpha}} n_\mu dx^\mu + \tilde{h}_{\tilde{\alpha}} \bar{m}_\mu dx^\mu \quad (1.10)$$

These four functions are the structure coordinates of the (integrable) complex structure used in (1.5). In the present case of Lorentzian spacetimes the coordinates $z^{\tilde{\alpha}}$ are not complex conjugate of z^α , because J_μ^ν is no longer a real tensor[3]. This peculiar property[9] was used by the author[11] to show that the particle spectrum of the present Lagrangian model is very rich, while the static potential of a source is no longer $\frac{1}{r}$ but it is linear.

A typical example of four dimensional complex structure compatible with the Minkowski metric are the light-cone coordinates determined by the following null tetrad

$$\begin{aligned} E_\mu^0 &\equiv L_\mu = \frac{1}{\sqrt{2}}(1, -1, 0, 0) \\ E_\mu^{\bar{0}} &\equiv N_\mu = \frac{1}{\sqrt{2}}(1, 1, 0, 0) \\ E_\mu^1 &\equiv M_\mu = \frac{1}{\sqrt{2}}(0, 0, 1, i) \\ E_\mu^{\bar{1}} &\equiv \bar{M}_\mu = \frac{1}{\sqrt{2}}(0, 0, 1, -i) \end{aligned} \quad (1.11)$$

which will be used in the present work. The general null tetrad will be expanded around this simple form.

In the case of the two-dimensional string action, no integrability conditions are required because any orientable two dimensional manifold is a complex manifold. But in the present case the conditions (1.9) must be introduced in the action using the Lagrange multiplier form

$$\begin{aligned} I_C = - \int d^4x \{ &\phi_0(\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu \ell_\nu) + \\ &+ \phi_1(\ell^\mu m^\nu - \ell^\nu m^\mu)(\partial_\mu m_\nu) + \phi_{\bar{0}}(n^\mu \bar{m}^\nu - n^\nu \bar{m}^\mu)(\partial_\mu n_\nu) + \\ &+ \phi_{\bar{1}}(n^\mu \bar{m}^\nu - n^\nu \bar{m}^\mu)(\partial_\mu \bar{m}_\nu) + c.conj. \} \end{aligned} \quad (1.12)$$

The complete action $I = I_G + I_C$ is self-consistent and it was quantized using the canonical (Dirac) quantization technique[8] and the path integral (BRST) technique[10].

The local symmetries of the action are a) the well known local gauge transformations, b) the reparametrization symmetry as it is the case in any generally covariant action and c) the following extended Weyl transformation of the tetrad

$$\begin{aligned} \ell'_\mu &= \chi_1 \ell_\mu \quad , \quad n'_\mu = \chi_2 n_\mu \quad , \quad m'_\mu = \chi m_\mu \\ \phi'_0 &= \phi_0 \frac{\chi_2 \bar{\chi}}{\chi_1} \quad , \quad \phi'_1 = \phi_1 \frac{\chi_2 \bar{\chi}}{\chi} \\ \phi'_{\bar{0}} &= \phi_{\bar{0}} \frac{\chi_1 \chi}{\chi_2} \quad , \quad \phi'_{\bar{1}} = \phi_{\bar{1}} \frac{\chi_1 \chi}{\bar{\chi}} \\ g' &= g(\chi_1 \chi_2 \chi \bar{\chi})^2 \end{aligned} \quad (1.13)$$

where χ_1, χ_2 are real functions and χ is a complex one.

2 GAUGE FIELD PROPAGATOR IN THE LANDAU AND FEYNMAN GAUGES

The enhanced conformal symmetry makes the present action unique and as far as I know the field propagator has never been considered in the literature. Therefore the gauge field propagator will be computed in the well known Landau and Feynman gauges, in order to familiarize the reader with the peculiarities

of the present gauge field action. A more appropriate gauge field condition will be used in the present work and the path integral (BRST) quantization[10] in this gauge will be described in the next sections. As usual the Feynman and Landau gauges are introduced in the path integral quantization through a term $\frac{1}{\alpha}(\eta^{\mu\nu}\partial_\mu A_{j\nu})^2$ in the effective action. The choices $\alpha = 1$ or $\alpha = 0$ are referred as Feynman and Landau gauges respectively. Following the well known path integral technique, the gauge field propagator (for arbitrary α) is

$$\langle TA_{i\mu}(x)A_{j\nu}(y)\rangle = -i\delta_{ij}\int\frac{d^4k}{(2\pi)^4}e^{ik(y-x)}\Delta_{\mu\nu}(k) \quad (2.1)$$

where $\Delta_{\mu\nu}(k)$ satisfies the relation

$$\begin{aligned} & [(L^\rho M^\mu - L^\mu M^\rho)(N^\lambda \overline{M}^\nu - N^\nu \overline{M}^\lambda) + (N^\rho \overline{M}^\mu - N^\mu \overline{M}^\rho)(L^\lambda M^\nu - L^\nu M^\lambda) + \\ & + (L^\rho \overline{M}^\mu - L^\mu \overline{M}^\rho)(N^\lambda M^\nu - N^\nu M^\lambda) + (N^\rho M^\mu - N^\mu M^\rho)(L^\lambda \overline{M}^\nu - L^\nu \overline{M}^\lambda) - \\ & - \frac{1}{\alpha}\eta^{\rho\mu}\eta^{\lambda\nu}]k_\rho k_\lambda \Delta_{\mu\nu}(k) = -\delta_\sigma^\mu \end{aligned} \quad (2.2)$$

which is found after the expansion of the action around the light-cone (integrable) null tetrad (1.11). Throughout this work the general null tetrad will be expanded around the light-cone one, because the calculations are highly simplified.

Expanding $\Delta_{\nu\sigma}(k)$ in this null tetrad

$$\begin{aligned} \Delta_{\nu\sigma} = & H_{00}L_\nu L_\sigma + H_{01}(L_\nu N_\sigma + L_\sigma N_\nu) + H_{02}(L_\nu M_\sigma + L_\sigma M_\nu) + \\ & + \overline{H}_{02}(L_\nu \overline{M}_\sigma + L_\sigma \overline{M}_\nu) + H_{11}N_\nu N_\sigma + H_{12}(N_\nu M_\sigma + N_\sigma M_\nu) + \\ & + \overline{H}_{12}(N_\nu \overline{M}_\sigma + N_\sigma \overline{M}_\nu) + H_{22}M_\nu M_\sigma + \\ & + H_{23}(M_\nu \overline{M}_\sigma + M_\sigma \overline{M}_\nu) + \overline{H}_{22}\overline{M}_\nu \overline{M}_\sigma \end{aligned} \quad (2.3)$$

and substituting into the above relation (2.2), a system of linear equations is derived, which can be directly solved. The final result is

$$\begin{aligned} H_{00} &= \frac{(Nk)(Nk)}{2(Mk)(\overline{M}k)k^2} + \frac{(\alpha-1)(Nk)(Nk)}{k^4} \\ H_{01} &= \frac{1}{k^2} \left[1 - \frac{(Lk)(Nk)}{2(Mk)(\overline{M}k)k^2} + \frac{(\alpha-1)(Lk)(Nk)}{k^2} \right] \\ H_{02} &= \frac{(1-\alpha)(Nk)(\overline{M}k)}{k^4} \\ H_{11} &= \frac{(Lk)(\overline{L}k)}{2(Mk)(\overline{M}k)k^2} + \frac{(\alpha-1)(Lk)(Lk)}{k^4} \\ H_{12} &= \frac{(1-\alpha)(Lk)(\overline{M}k)}{k^4} \\ H_{22} &= -\frac{(\overline{M}k)(\overline{M}k)}{2(Lk)(Nk)k^2} + \frac{(\alpha-1)(\overline{M}k)(\overline{M}k)}{k^4} \\ H_{23} &= \frac{1}{k^2} \left[-1 + \frac{(Mk)(\overline{M}k)}{2(Lk)(Nk)} + \frac{(\alpha-1)(Mk)(\overline{M}k)}{k^4} \right] \end{aligned} \quad (2.4)$$

where the short notation $(E_a k) \equiv E_a^\mu k_\mu$ is used. In fact these are the light-cone coordinates of the four-vector k_μ . This short light-cone notation will be used throughout this work in order to keep track of the initial tetrad structure of the different Lagrangian terms..

In the Landau gauge ($\alpha = 0$) the Fourier transform of the gauge field propagator takes the form

$$\begin{aligned} \langle T A_{i\mu}(x) A_{j\nu}(y) \rangle_F = & -\frac{i\delta_{ij}}{k^2} [\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} + \frac{(Nk)(Nk)}{2(Mk)(\overline{M}k)} L_\mu L_\nu + \\ & + \frac{(Lk)(Lk)}{2(Mk)(\overline{M}k)} N_\mu N_\nu - \frac{(Lk)(Nk)}{2(Mk)(\overline{M}k)} (L_\mu N_\nu + L_\nu N_\mu) - \frac{(\overline{M}k)(\overline{M}k)}{2(Lk)(Nk)} M_\mu M_\nu + \\ & + \frac{(Mk)(\overline{M}k)}{2(Lk)(Nk)} (M_\mu \overline{M}_\nu + M_\nu \overline{M}_\mu) - \frac{(Mk)(Mk)}{2(Lk)(Nk)} \overline{M}_\mu \overline{M}_\nu] \end{aligned} \quad (2.5)$$

Notice that in addition to the ordinary term $\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$ of the gauge field propagator it contains non-conventional terms too. The difference of the present gauge field action (1.7) from the ordinary one appears in the gauge field propagator too, as we should expect. In the Feynman gauge ($\alpha = 1$) only the ordinary part of the propagator changes to the well known form. The additional non-conventional terms remain the same.

3 AN APPROPRIATE GAUGE CONDITION

In the Landau and Feynman gauges, the gauge field propagators are very complicated. Therefore they are not convenient for the computation of the Feynman diagrams. It was found that the most convenient gauge condition is

$$M^\mu \partial_\mu (\overline{M} A_j) + \overline{M}^\mu \partial_\mu (M A_j) = 0 \quad (3.1)$$

where $(E^a A_j) \equiv E^{a\mu} A_{j\mu}$ are the light-cone coordinates of the gauge field $A_{j\mu}$. In section 7 explicit calculations of the first order one-loop diagrams will be performed. The great advantage of this precise gauge condition is that these diagrams are found to be finite. That is no counterterms appear!

In the path integral formulation, the validity of a gauge condition is formally assured through the non-annihilation of the Faddeev-Popov determinant. It will be checked below in the case of an Abelian $U(1)$ gauge field. It is generally assumed that the same results are perturbatively extended to the non-Abelian cases modulo possible Gribov ambiguities. The above gauge condition yields the following Faddeev-Popov operator

$$M_{FP} = - \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right). \quad (3.2)$$

The determinant of this operator does not vanish, because it has no regular asymptotically vanishing eigenfunction with zero eigenvalue. One can see it by simply writing this operator in polar coordinates and making a Fourier expansion. Then we see that the zero modes must satisfy the following differential equation

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{n^2}{\rho^2} \right) \Lambda_n(t, x, \rho) = 0 \quad (3.3)$$

For $n \neq 0$ the general solution of this equation is

$$\Lambda_n(t, x, \rho) = h_{1n}(t, x) \rho^n + h_{2n}(t, x) \rho^{-n} \quad (3.4)$$

which is regular at $\rho = 0$ if $h_2 = 0$ and it vanishes at infinity if $h_1 = 0$. For $n = 0$ the solution is

$$\Lambda_0(t, x, \rho) = h_{10}(t, x) + h_{20}(t, x) \ln \rho \quad (3.5)$$

which does not satisfy the regularity conditions. Hence we see that the kernel of the Faddeev-Popov operator contains only the zero function.

One should not be confused by the apparent permitted gauge transformation

$$A'_\mu = A_\mu - \partial_\mu \Lambda(t, x) \quad (3.6)$$

because the asymptotic annihilation is assumed in all space directions. $\Lambda(t, x)$ must vanish because at ρ -infinity it is the same function. Recall that the same argument is applied to the case of the axial gauge condition of the electromagnetic field too.

In the conventional procedure, the non-vanishing of the Faddeev-Popov determinant means that the gauge condition uniquely fixes the gauge freedom of the action. The additional point, one should clarify, is that the precise gauge can always be reached starting from any regular asymptotically vanishing field configuration $A_\mu(x)$. One can see that it is reachable, if there is a regular asymptotically vanishing solution of the differential equation

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Lambda = M^\mu \partial_\mu (\overline{M} A_j) + \overline{M}^\mu \partial_\mu (M A_j) \equiv f(x) \quad (3.7)$$

In polar coordinates and after a Fourier expansion it becomes the following ordinary differential equation

$$\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} - \frac{n^2}{\rho^2} \right) \Lambda_n(t, x, \rho) = f_n(t, x, \rho) \quad (3.8)$$

which always admits a solution with initial conditions

$$\Lambda_n(t, x, 0) = 0 \quad , \quad \frac{d\Lambda_n}{d\rho}(t, x, 0) = 0 \quad (3.9)$$

The above analysis of the convenient gauge condition shows that it is well defined and it may be used to determine the gauge field propagator.

4 LAGRANGIAN EXPANSION AND PROPAGATORS

The Dirac[8] and BRST[10] quantizations of the model will be used to study its renormalizability. The path-integral (BRST) quantization can be accomplished by simply following the ordinary steps. We first see that the local symmetries of the complete action are the usual gauge symmetry, reparametrization and the extended Weyl transformations. For every local symmetry we have to assume a

gauge condition. Here we must be careful to impose convenient gauge conditions such that the induced Faddeev-Popov determinant to have vanishing the upper diagonal elements in order to be reduced down into the product of the three determinants which correspond to the three local symmetries of the action. The gauge symmetry is fixed using the appropriate gauge condition (3.1). The additional extended Weyl symmetry of the tetrad is fixed using the following conditions

$$\begin{aligned} \ell^\mu N_\mu - 1 = 0 \quad , \quad n^\mu L_\mu - 1 = 0 \\ \overline{m}^\mu M_\mu + 1 = 0 \quad , \quad m^\mu \overline{M}_\mu + 1 = 0 \end{aligned} \quad (4.1)$$

The convenient conditions which fix the reparametrization symmetry are

$$\begin{aligned} L^\mu \ell_\mu n^\nu L_\nu = 0 \quad , \quad N^\mu n_\mu \ell^\nu N_\nu = 0 \\ M^\mu m_\mu \overline{m}^\nu M_\nu = 0 \quad , \quad \overline{M}^\mu \overline{m}_\mu m^\nu \overline{M}_\nu = 0 \end{aligned} \quad (4.2)$$

Then the Faddeev-Popov terms of the effective Lagrangian are the following[10]

$$\begin{aligned} I_{FP} = \int d^4x \{ & -\frac{1}{2\alpha} [M^\mu \partial_\mu (\overline{M} A_j) + \overline{M}^\mu \partial_\mu (M A_j)]^2 + B_1 (\ell^\mu N_\mu - 1) + \\ & + B_2 (n^\mu L_\mu - 1) + B_3 (\overline{m}^\mu M_\mu + 1) + B_4 (m^\mu \overline{M}_\mu + 1) + \\ & + B_5 (L^\mu \ell_\mu) (n^\nu L_\nu) + B_6 (N^\mu n_\mu) (\ell^\nu N_\nu) + \\ & + B_7 (M^\mu m_\mu) (\overline{m}^\nu M_\nu) + B_8 (\overline{M}^\mu \overline{m}_\mu) (m^\nu \overline{M}_\nu) + \\ & + M^\mu (\partial_\mu \overline{d}_j) [\overline{M}^\nu (\partial_\nu d_j) - q f_{jik} d_i (\overline{M}^\nu A_{k\nu})] \\ & + \overline{M}^\mu (\partial_\mu \overline{d}_j) [M^\nu (\partial_\nu d_j) - q f_{jik} d_i (M^\nu A_{k\nu})] \\ & - \overline{c}_1 L^\mu [c^\nu (\partial_\nu \ell_\mu) + \ell_\nu (\partial_\mu c^\nu)] - \overline{c}_2 N^\mu [c^\nu (\partial_\nu n_\mu) + n_\nu (\partial_\mu c^\nu)] - \\ & - \overline{c}_3 M^\mu [c^\nu (\partial_\nu m_\mu) + m_\nu (\partial_\mu c^\nu)] - \overline{c}_4 \overline{M}^\mu [c^\nu (\partial_\nu \overline{m}_\mu) + \overline{m}_\nu (\partial_\mu c^\nu)] \} \end{aligned} \quad (4.3)$$

where \overline{d}_j and d_j are the ghost fields which correspond to the gauge field condition and \overline{c}_i , c_i are the ghost fields which correspond to the reparametrization symmetry. The extended Weyl symmetry on the tetrad does not generate any ghost field.

In order to compute the Feynman diagrams we have first to expand the Lagrangian around a classical solution of the field equations. In the present case it is convenient to expand the general null tetrad around the trivial light-cone tetrad E_a^μ that we have chosen to introduce the conditions which fix the reparametrization and Weyl symmetries. That is, we consider the expansion

$$\begin{aligned} \ell^\mu &= L^\mu + \gamma \varepsilon_0^\mu \\ n^\mu &= N^\mu + \gamma \varepsilon_0^\mu \\ m^\mu &= M^\mu - \gamma \varepsilon_1^\mu \end{aligned} \quad (4.4)$$

where γ is a dimensionless constant. Notice that in the Lagrangian there is no dimensional constant, which could generate non-renormalizable counterterms through the regularization procedure. In this tetrad expansion, the conditions become

$$\begin{aligned} \varepsilon_0^\mu N_\mu = 0 \quad , \quad \varepsilon_0^\mu L_\mu = 0 \quad , \quad \varepsilon_1^\mu M_\mu = 0 \\ \varepsilon_0^\mu L_\mu - \gamma [(\varepsilon_0^\nu M_\nu)(\varepsilon_1^\rho L_\rho) + (\varepsilon_0^\nu \overline{M}_\nu)(\varepsilon_1^\rho L_\rho)] + O(\gamma^2) = 0 \\ \varepsilon_0^\mu N_\mu - \gamma [(\varepsilon_0^\nu M_\nu)(\varepsilon_1^\rho N_\rho) + (\varepsilon_0^\nu \overline{M}_\nu)(\varepsilon_1^\rho N_\rho)] + O(\gamma^2) = 0 \\ \varepsilon_1^\mu \overline{M}_\mu - \gamma [(\varepsilon_1^\nu L_\nu)(\varepsilon_0^\rho \overline{M}_\rho) + (\varepsilon_1^\nu N_\nu)(\varepsilon_0^\rho \overline{M}_\rho)] + O(\gamma^2) = 0 \end{aligned} \quad (4.5)$$

They can be solved and replaced back into the action, which is so expanded in the dimensionless coupling constants γ and q . The first terms of this expansion of the I_G part of the action are the following

$$\begin{aligned}
I_G \simeq \int d^4x \{ & [(LM\partial A_j)(N\overline{M}\partial A_j) + (L\overline{M}\partial A_j)(NM\partial A_j)] - \\
& - qf_{jik}[(LA_i)(MA_k)(N\overline{M}\partial A_j) + (NA_i)(\overline{M}A_k)(LM\partial A_j) + c.c.] + \\
& + \gamma[(M\varepsilon_{\tilde{0}})(M\overline{M}\partial A_j)(N\overline{M}\partial A_j) - (L\varepsilon_{\tilde{1}})(LN\partial A_j)(N\overline{M}\partial A_j) + \\
& + (N\varepsilon_1)(LM\partial A_j)(LN\partial A_j) - (\overline{M}\varepsilon_0)(LM\partial A_j)(M\overline{M}\partial A_j) + c.c.] + \\
& + q^2 f_{jik} f_{ji'k'} [(LA_i)(MA_k)(NA_{i'}) (\overline{M}A_{k'}) + c.c.] \}
\end{aligned} \tag{4.6}$$

where short notations of the form $(LM\partial A_j) = (L^\mu M^\nu - L^\nu M^\mu)(\partial_\mu A_{j\nu})$ etc are used in order to simplify the appearance of this and the following expressions. The first terms of the I_C part of the action are

$$\begin{aligned}
I_C \simeq \int d^4x \{ & -[\phi_0 L^\nu \partial_\nu (L\varepsilon_{\tilde{1}}) + \phi_1 M^\nu \partial_\nu (M\varepsilon_{\tilde{0}}) + \\
& + \phi_{\tilde{0}} N^\nu \partial_\nu (N\varepsilon_1) + \phi_{\tilde{1}} \overline{M}^\nu \partial_\nu (\overline{M}\varepsilon_0) + c.c.] - \\
& - \gamma[\phi_0 (M\varepsilon_{\tilde{0}})[M^\nu \partial_\nu (L\varepsilon_1) - \overline{M}^\nu \partial_\nu (L\varepsilon_{\tilde{1}})] + \\
& + \phi_1 (L\varepsilon_{\tilde{1}})[L^\nu \partial_\nu (M\varepsilon_0) - N^\nu \partial_\nu (M\varepsilon_{\tilde{0}})] + \\
& + \phi_{\tilde{0}} (\overline{M}\varepsilon_0)[\overline{M}^\nu \partial_\nu (N\varepsilon_{\tilde{1}}) - M^\nu \partial_\nu (N\varepsilon_1)] + \\
& + \phi_{\tilde{1}} (N\varepsilon_1)[N^\nu \partial_\nu (\overline{M}\varepsilon_{\tilde{0}}) - L^\nu \partial_\nu (\overline{M}\varepsilon_0)] + c.c.] \}
\end{aligned} \tag{4.7}$$

The first terms of the I_{FP} part of the action are

$$\begin{aligned}
I_{FP} \simeq \int d^4x \{ & -\frac{1}{2\alpha} [M^\mu \partial_\mu (\overline{M}A_j) + \overline{M}^\mu \partial_\mu (MA_j)]^2 - \\
& - 2\overline{d}_j M^\mu \overline{M}^\mu (\partial_\mu \partial_\nu d_j) - \overline{c}_1 L^\mu \partial_\mu (Lc) - \\
& - \overline{c}_2 N^\mu \partial_\mu (Nc) - \overline{c}_3 M^\mu \partial_\mu (Mc) - \overline{c}_4 \overline{M}^\mu \partial_\mu (\overline{M}c) - \\
& - qf_{jik} [M^\mu (\partial_\mu \overline{d}_j) d_i (\overline{M}A_k) + \overline{M}^\mu (\partial_\mu \overline{d}_j) d_i (MA_k)] + \\
& + \gamma [\overline{c}_1 (L\varepsilon_1) L^\mu \partial_\mu (Mc) + \overline{c}_1 (L\varepsilon_{\tilde{1}}) L^\mu \partial_\mu (\overline{M}c) + \\
& + \overline{c}_2 (N\varepsilon_1) N^\mu \partial_\mu (Mc) + \overline{c}_2 (N\varepsilon_{\tilde{1}}) N^\mu \partial_\mu (\overline{M}c) + \\
& + \overline{c}_3 (M\varepsilon_0) N^\mu \partial_\mu (Lc) + \overline{c}_3 (M\varepsilon_{\tilde{0}}) M^\mu \partial_\mu (Nc) + \\
& + \overline{c}_4 (\overline{M}\varepsilon_0) \overline{M}^\mu \partial_\mu (Lc) + \overline{c}_4 (\overline{M}\varepsilon_{\tilde{0}}) \overline{M}^\mu \partial_\mu (Nc)] \}
\end{aligned} \tag{4.8}$$

where the already defined short light-cone notation is used.

The zeroth order terms of this action expansion determine the field propagators. The Fourier transforms of the gauge field propagator has the following for general α

$$\begin{aligned}
\langle T A_{i\mu}(x) A_{j\nu}(y) \rangle_F = & -\frac{i\delta_{ij}}{4(Mk)(\overline{M}k)} \left[\frac{\alpha(Nk)(Nk)}{(Mk)(\overline{M}k)} L_\mu L_\nu + \frac{\alpha(Lk)(Lk)}{(Mk)(\overline{M}k)} N_\mu N_\nu + \right. \\
& + \frac{\alpha(Lk)(Nk) - 2(Mk)(\overline{M}k)}{(Mk)(\overline{M}k)} (L_\mu N_\nu + L_\nu N_\mu) - \frac{\alpha(Nk)(\overline{M}k)}{(Mk)(\overline{M}k)} (L_\mu M_\nu + L_\nu M_\mu) - \\
& - \frac{\alpha(Nk)(Mk)}{(Mk)(\overline{M}k)} (L_\mu \overline{M}_\nu + L_\nu \overline{M}_\mu) - \frac{\alpha(Lk)(\overline{M}k)}{(Mk)(\overline{M}k)} (N_\mu M_\nu + N_\nu M_\mu) - \\
& - \frac{\alpha(Lk)(Mk)}{(Mk)(\overline{M}k)} (L_\mu \overline{M}_\nu + L_\nu \overline{M}_\mu) + \frac{\alpha(Lk)(Nk) + (Mk)(\overline{M}k)}{(Lk)(Nk)(Mk)(\overline{M}k)} M_\mu M_\nu - \\
& \left. + \frac{\alpha(Lk)(Nk) - (Mk)(\overline{M}k)}{(Lk)(Nk)} (M_\mu \overline{M}_\nu + M_\nu \overline{M}_\mu) + \frac{(Mk)(Mk)(\alpha(Lk)(Nk) + (Mk)(\overline{M}k))}{(Lk)(Nk)(Mk)(\overline{M}k)} \overline{M}_\mu \overline{M}_\nu \right]
\end{aligned} \tag{4.9}$$

One can easily find that in the special gauge $\alpha = 0$ the non-vanishing terms of gauge field propagator are

$$\begin{aligned}
\langle T(LA_i)(NA_j) \rangle_F &= \frac{i\delta_{ij}}{2(Mk)(\overline{M}k)} \\
\langle T(MA_i)(MA_j) \rangle_F &= -\frac{i(Mk)(\overline{M}k)\delta_{ij}}{4(Lk)(Nk)(Mk)(\overline{M}k)} \\
\langle T(\overline{M}A_i)(\overline{M}A_j) \rangle_F &= -\frac{i(\overline{M}k)(Mk)\delta_{ij}}{4(Lk)(Nk)(Mk)(\overline{M}k)} \\
\langle T(MA_i)(\overline{M}A_j) \rangle_F &= \frac{i\delta_{ij}}{4(Lk)(Nk)}
\end{aligned} \tag{4.10}$$

where the previously defined light-cone short notation is used

$$\begin{aligned}
(Lk) &= \frac{k^0 - k^1}{\sqrt{2}} \\
(Nk) &= \frac{k^0 + k^1}{\sqrt{2}} \\
(Mk) &= \frac{k^2 + ik^3}{\sqrt{2}}
\end{aligned} \tag{4.11}$$

Notice that this propagator is essentially the product of two well known 2-dimensional scalar field propagator

$$D_{L(E)} = \int \frac{d^2k}{(2\pi)^2} \frac{e^{ikx}}{k^2 + i\varepsilon} = \frac{i}{4\pi} \int \frac{dt}{t} e^{-i(x^2 - i\varepsilon)t} \tag{4.12}$$

where the indices L and E correspond to the signatures $(+, -)$ and $(-, -)$ respectively. This propagator is logarithmically divergent, but the difference $D(x) - D(x_0)$ is apparently finite. One can easily find that the explicit form of the present gauge field propagator is

$$\begin{aligned}
\langle T(LA_i(0))(NA_j(x)) \rangle &= -i\delta_{ij}\delta(x^0)\delta(x^1)D_E(x^2, x^3) \\
\langle T(MA_i(0))(MA_j(x)) \rangle &= i\delta_{ij}D_L(x^0, x^1)M^\mu M^\nu \partial_\mu \partial_\nu D_E(x^2, x^3) \\
\langle T(\overline{M}A_i(0))(\overline{M}A_j(x)) \rangle &= i\delta_{ij}D_L(x^0, x^1)\overline{M}^\mu \overline{M}^\nu \partial_\mu \partial_\nu D_E(x^2, x^3) \\
\langle T(MA_i(0))(\overline{M}A_j(x)) \rangle &= i\delta_{ij}D_L(x^0, x^1)\delta(x^2)\delta(x^3)
\end{aligned} \tag{4.13}$$

The Fourier transforms of the other field propagators are

$$\begin{aligned}
\langle T\phi_0(L\varepsilon_1) \rangle_F &= -\frac{1}{(Lk)} \quad , \quad \langle T\phi_1(M\varepsilon_0) \rangle_F = -\frac{1}{(Mk)} \\
\langle T\phi_0(N\varepsilon_1) \rangle_F &= -\frac{1}{(Nk)} \quad , \quad \langle T\phi_1(\overline{M}\varepsilon_0) \rangle_F = -\frac{1}{(\overline{M}k)} \\
\langle T\overline{c}_1(Lc) \rangle_F &= \frac{1}{(Lk)} \quad , \quad \langle T\overline{c}_2(Nc) \rangle_F = \frac{1}{(Nk)} \\
\langle T\overline{c}_3(Mc) \rangle_F &= \frac{1}{(\overline{M}k)} \quad , \quad \langle T\overline{c}_4(\overline{M}c) \rangle_F = \frac{1}{(Mk)} \\
\langle Td_i\overline{d}_j \rangle_F &= \frac{i\delta_{ij}}{2(Mk)(\overline{M}k)}
\end{aligned} \tag{4.14}$$

Notice that there is no tetrad-tetrad propagator. Only ϕ_b -tetrad propagators exist. This implies that there is no loop diagram with ϕ_b external lines. The one-particle irreducible (1PI) diagrams of the model do not contain ϕ - ε and \overline{c} - c propagators. This crucial property implies that there is no divergent candidate to renormalize the term I_C of the action. Hence the regularization procedure does not affect the integrability of the complex structure and subsequently the metric independence of the action in a structure coordinate neighborhood.

5 RENORMALIZABILITY

The present action does not contain any dimensional parameter like the ordinary gauge field action. Therefore the dimensionality of the counterterms will be four. It also admits the enhanced Weyl symmetry (1.13), which does not permit the counterterm (1.2) with the Weyl tensor. This means that no pure metric dependent action counterterm will be generated. The action has also the following discrete symmetry

$$\begin{aligned} a) \ell^\mu &\Leftrightarrow n^\mu, & \phi_0 &\Leftrightarrow \overline{\phi_0}, & \phi_1 &\Leftrightarrow \overline{\phi_1} \\ b) m^\mu &\Leftrightarrow \overline{m}^\mu, & \phi_a &\Leftrightarrow \overline{\phi_a}, & \forall a \end{aligned} \quad (5.1)$$

which assures that the extended Weyl preserving term $\sqrt{-g}(\ell^\mu n^\rho F_{j\mu\rho})(m^\nu \overline{m}^\sigma F_{j\nu\sigma})$ does not emerge from the renormalization procedure.

One might think that the integrability condition of the complex structure is not necessary for the renormalizability of the action. This is not true, because the mentioned cases do not exhaust all the possible counterterms. Notice that the action depends on the tetrad and not directly on the metric of the space-time. In fact no metric appears in the action of the model, where the tetrad vectors $(\ell_\mu, n_\mu, m_\mu, \overline{m}_\mu)$ must be treated as four independent vector fields. Therefore there may be generally covariant tetrad terms invariant relative to the extended Weyl symmetry and the above discrete symmetry. The following 1-dimensionality forms and their complex conjugate transform as densities relative to the extended Weyl symmetry.

$$\begin{aligned} &(\ell n \partial m), (\ell m \partial \ell), (\ell m \partial m) \\ &(n m \partial n), (n m \partial m), (m \overline{m} \partial \ell), (m \overline{m} \partial n) \end{aligned} \quad (5.2)$$

Where the compact notation $(\ell n \partial m)$ has already defined and it denotes $(\ell n \partial m) = (\ell^\mu n^\nu - \ell^\nu n^\mu)(\partial_\mu m_\nu)$. They may be combined to generate invariant polynomial and/or non polynomial Lagrangian terms. A typical example of a 4-dimensional Lagrangian symmetric term is the following

$$\int d^4x \sqrt{-g} (\ell n \partial m) (\ell n \partial \overline{m}) (m \overline{m} \partial \ell) (m \overline{m} \partial n) \quad (5.3)$$

Notice that this term is not affected (annihilated) by the integrability conditions of the complex structure. Therefore in principle such a counterterm could be generated. Its exclusion is implied by the following argument.

The complex structure integrability conditions give the present action the form (1.5). It is apparently tetrad independent. This means that there is a coordinate system where the action is tetrad independent. Therefore any geometric term (tetrad dependent) cannot be generated as long as the renormalization procedure does not change the action term (1.12) which imposes the complex structure integrability conditions. This is valid, as it has already been pointed out at the end of the previous section, because there is no tetrad-tetrad propagator. It implies that there is no one particle irreducible loop diagram with ϕ external lines. Hence the action term (1.12) is not affected by the renormalization procedure neither other terms with ϕ factors can emerge.

6 REGULARIZATION

The expansion around the constant light-cone tetrad separates the 4-dimensional spacetime into two different 2-dimensional spaces, because in the convenient gauge condition all the field propagators become the product of two 2-dimensional propagators or one 2-dimensional propagator and a 2-dimensional delta function. This is the characteristic property of the special gauge condition which is responsible for the finiteness of the loop diagrams computed below. Any loop-integral turns out to become the product of two independent 2-dimensional integrals. Therefore the dimensional regularization must be simultaneously performed in both 2-dimensional subspaces. It is done by extending the dimension of the (L_μ, N_μ) -subspace into 2ω and the dimension of the $(M_\mu, \overline{M}_\mu)$ -subspace into $2\omega'$.

When the dimension of the spacetime changes into $2(\omega + \omega')$ the number of tetrads changes too. Therefore we first make the substitutions $2(Lk)(Nk) = k^2$ and $2(Mk')(\overline{M}k') = k'^2$ in all the integrals and after they are dimensionally regularized. The results are finally contracted with the remaining tetrads using the formula

$$E_a^\mu E_b^\nu \eta_{\mu\nu} = \eta_{ab} \quad (6.1)$$

which does not contain the spacetime dimension. It does appear after the additional contraction with η^{ab} .

The formula of the dimensional regularization, which will be applied are the called “’t Hooft-Veltman conjecture”[12]

$$\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} (k^2)^{\beta-1} = 0 \quad \forall \beta = 0, 1, 2, \dots \quad (6.2)$$

and the following logarithmically divergent 2-dimensional integral

$$I_{\rho\nu} = \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{k_\rho k_\nu}{k^2(k-p)^2} = \eta_{\rho\nu} \frac{\Gamma(1-\omega)}{2(4\pi)^\omega} \int_0^1 dx [x(1-x)p^2 + \mu^2]^{\omega-1} + p_\rho p_\nu \frac{\Gamma(2-\omega)}{(4\pi)^\omega} \int_0^1 dx x^2 [x(1-x)p^2 + \mu^2]^{\omega-2} \quad (6.3)$$

where the ordinary mass term μ^2 has been introduced in order to distinguish the ultraviolet from the infrared divergencies. Notice that in the infrared limit ($\mu^2 = 0$) the annihilation of the tadpole diagram ($\beta = 0$ in the ’t Hooft-Veltman conjecture) is rederived[4].

In the present 2-dimensional case ($\omega = 1$) the second term of $I_{\rho\nu}$ has no ultraviolet divergence, therefore the following integrals, which appear in the calculations, are finite.

$$\begin{aligned} \int \frac{d^2k}{(2\pi)^2} \frac{(Lk)}{(Nk)(L \cdot (k-p))(N \cdot (k-p))} &= i(Lp)^2 \int_0^1 dx \frac{x^2}{x(1-x)(-p^2) + \mu^2} \\ \int \frac{d^2k'}{(2\pi)^2} \frac{(Mk')}{(\overline{M}k')(M \cdot (k'-p'))(\overline{M} \cdot (k'-p'))} &= (Mp)^2 \int_0^1 dx \frac{x^2}{x(1-x)(p'^2) + \mu'^2} \end{aligned} \quad (6.4)$$

where no-primed k, p denote the (L_μ, N_μ) -subspace and the primed k', p' denote the $(M_\mu, \overline{M}_\mu)$ -subspace components of the 4-momenta k, p . Analogous results are found in the $(N^\mu N^\nu I_{\mu\nu})$ and $(\overline{M}^\mu \overline{M}^\nu I_{\mu\nu})$ contractions.

7 FIRST ORDER ONE-LOOP DIAGRAMS ARE FINITE

It has already been stated that there are no loop diagrams with $\phi_a(x)$ external lines. The three possible cases of first order one-loop diagrams are a) with external tetrads and b) with external gauge fields. I find more convenient to use the Bogolioubov-Chirkov procedure[1] for the computation of the S-matrix one-loop terms as time-ordered products. Only the main points will be outlined, because it is practically impossible to present all the calculations here.

a) *Diagrams with two external tetrads.* These diagrams come from the contractions between internal couplings of I_G , I_C and I_{FP} separately. The ghost field contractions give

$$\begin{aligned}
[2 \text{ ext. tetrads from } I_{FP}] = & -\gamma^2 \int d^4 y_1 d^4 y_2 \{ (L\varepsilon_1(1))(M\varepsilon_0(2)) : \\
& \cdot \langle T\bar{c}_1(1)M^\mu\partial_\mu(L_\nu c^\nu(2)) \rangle \langle TL^\mu\partial_\mu(M_\nu c^\nu(1))\bar{c}_3(2) \rangle + \\
& : (L_\rho\varepsilon_1^\rho(1))(\bar{M}_\tau\varepsilon_0^\tau(2)) : \langle T\bar{c}_1(1)\bar{M}^\mu\partial_\mu(L_\nu c^\nu(2)) \rangle \langle TL^\mu\partial_\mu(\bar{M}_\nu c^\nu(1))\bar{c}_4(2) \rangle + \\
& : (N_\rho\varepsilon_1^\rho(1))(M_\tau\varepsilon_0^\tau(2)) : \langle T\bar{c}_2(1)M^\mu\partial_\mu(N_\nu c^\nu(2)) \rangle \langle TN^\mu\partial_\mu(M_\nu c^\nu(1))\bar{c}_3(2) \rangle + \\
& : (N_\rho\varepsilon_1^\rho(1))(\bar{M}_\tau\varepsilon_0^\tau(2)) : \langle T\bar{c}_2(1)\bar{M}^\mu\partial_\mu(N_\nu c^\nu(2)) \rangle \langle TN^\mu\partial_\mu(\bar{M}_\nu c^\nu(1))\bar{c}_4(2) \rangle \}
\end{aligned} \tag{7.1}$$

where $: \dots :$ denotes the Wick product and the integration variables y_1, y_2 are briefly denoted 1 and 2 respectively.

After the substitution of the propagators and some well known changes of variables, (7.1) takes the following form

$$\begin{aligned}
[2 \text{ ext. tetrads from } I_{FP}] = & -\gamma^2 \int d^4 y_1 d^4 y_2 \{ (L_\rho\varepsilon_1^\rho(1))(M_\tau\varepsilon_0^\tau(2)) : \\
& \cdot \left[\int \frac{d^2 k}{(2\pi)^2} \frac{(L(p-k))}{(Lk)} \right] \left[\int \frac{d^2 k'}{(2\pi)^2} \frac{(M(p-k'))}{(Mk')} \right] + \\
& + : (L\varepsilon_1)(\bar{M}\varepsilon_0) : \left[\int \frac{d^2 k}{(2\pi)^2} \frac{(L(p-k))}{(Lk)} \right] \left[\int \frac{d^2 k'}{(2\pi)^2} \frac{(\bar{M}(p-k'))}{(\bar{M}k')} \right] + \\
& + : (N\varepsilon_1)(M\varepsilon_0) : \left[\int \frac{d^2 k}{(2\pi)^2} \frac{(N(p-k))}{(Nk)} \right] \left[\int \frac{d^2 k'}{(2\pi)^2} \frac{(M(p-k'))}{(Mk')} \right] + \\
& + : (N\varepsilon_1)(\bar{M}\varepsilon_0) : \left[\int \frac{d^2 k}{(2\pi)^2} \frac{(N(p-k))}{(Nk)} \right] \left[\int \frac{d^2 k'}{(2\pi)^2} \frac{(\bar{M}(p-k'))}{(\bar{M}k')} \right] \}
\end{aligned} \tag{7.2}$$

where the defined light-cone notation is used.

Using the formulas of the regularization subsection one can show that all the above integrals vanish in the context of the dimensional regularization.

The integrals generated by the I_C couplings are analogous to the previous ones and they are found to vanish too. The expression is too long to be written down here, therefore only the diagram with $(L\varepsilon_1)$ $(N\varepsilon_0)$ external lines will be presented in order to be shown how they look like.

$$\begin{aligned}
[(L\varepsilon_1)(N\varepsilon_0) \text{ from } I_H] = & -\gamma^2 \int d^4 y_1 d^4 y_2 : (L\varepsilon_1)(N\varepsilon_0) : \cdot \\
& \cdot \langle T\bar{\phi}_0 N^\mu\partial_\mu(\bar{M}\varepsilon_0) \rangle \langle TL^\nu\partial_\nu(\bar{M}\varepsilon_0)\phi_0 \rangle = \\
= & -\gamma^2 \int d^4 y_1 d^4 y_2 \{ (L\varepsilon_1)(N\varepsilon_0) : \int \frac{d^4 p}{(2\pi)^4} e^{ip(y_2-y_1)} \cdot \\
& \cdot \left[\int \frac{d^2 k}{(2\pi)^2} (Nk)(L(p-k)) \right] \left[\int \frac{d^2 k'}{(2\pi)^2} \frac{1}{(\bar{M}k')(\bar{M}(p-k'))} \right] \}
\end{aligned} \tag{7.3}$$

This term vanishes because of the 't Hooft-Veltman conjecture applied to the k -integration.

The diagrams from the I_G couplings, with gauge field contractions, are

$$\begin{aligned}
[2 \text{ ext. tetrads from } I_G] = & -\frac{\gamma^2}{2} \int d^4 y_1 d^4 y_2 \{ : (\overline{M}\varepsilon_0)(\overline{M}\varepsilon_0) : \cdot \\
& \cdot [\langle T(LM\partial A_j)(LM\partial A_k) \rangle \langle T(M\overline{M}\partial A_j)(M\overline{M}\partial A_k) \rangle + \\
& + \langle T(LM\partial A_j)(M\overline{M}\partial A_k) \rangle \langle T(M\overline{M}\partial A_j)(LM\partial A_k) \rangle] - \\
& - 2 : (\overline{M}\varepsilon_0)(N\varepsilon_1) : [\langle T(LM\partial A_j)(LM\partial A_k) \rangle \langle T(M\overline{M}\partial A_j)(LN\partial A_k) \rangle + \\
& + \langle T(LM\partial A_j)(LN\partial A_k) \rangle \langle T(M\overline{M}\partial A_j)(LM\partial A_k) \rangle] - \\
& - 2 : (M\varepsilon_0)(N\varepsilon_0) : [\langle T(LM\partial A_j)(M\overline{M}\partial A_k) \rangle \langle T(M\overline{M}\partial A_j)(N\overline{M}\partial A_k) \rangle + \\
& + \langle T(LM\partial A_j)(N\overline{M}\partial A_k) \rangle \langle T(M\overline{M}\partial A_j)(M\overline{M}\partial A_k) \rangle] + \text{similar terms} \}
\end{aligned} \tag{7.4}$$

This expression is also too long to be written down. I computed all these integrals and I found that they vanish. The conclusion is that there is no counterterm with two external tetrads.

b) Diagrams with external gauge fields. The number of these diagrams is quite large, but they can be grouped using the discrete symmetries (5.1) of the action. The diagrams with $(LA_i)(LA_j)$ external terms give

$$\begin{aligned}
[\text{ext}(LA_i)(LA_j)] = & -\frac{\gamma^2}{2} \int d^4 y_1 d^4 y_2 f_{j_1 i_1 k_1} f_{j_2 i_2 k_2} : (LA_{i_1})(LA_{i_2}) : \cdot \\
& \cdot [\langle T(MA_{k_1})(MA_{k_2}) \rangle \langle T(N\overline{M}\partial A_{j_1})(N\overline{M}\partial A_{j_2}) \rangle + \\
& + \langle T(MA_{k_1})(N\overline{M}\partial A_{j_2}) \rangle \langle T(N\overline{M}\partial A_{j_1})(MA_{k_2}) \rangle + \\
& + \langle T(MA_{k_1})(\overline{M}A_{k_2}) \rangle \langle T(N\overline{M}\partial A_{j_1})(NM\partial A_{j_2}) \rangle + \\
& + \langle T(MA_{k_1})(NM\partial A_{j_2}) \rangle \langle T(N\overline{M}\partial A_{j_1})(\overline{M}A_{k_2}) \rangle + c.c.] = \\
= & -\frac{i\gamma^2 C}{16(4\pi)^2} \int d^4 y_1 d^4 y_2 \int \frac{d^4 p}{(2\pi)^4} e^{ip(y_2 - y_1)} : (LA_{i_1})(LA_{i_2}) : \cdot \\
& \cdot \delta_{i_1 i_2} (Np)^2 (Mp)^2 (\overline{M}p)^2 I_1(p'^2) [2I_2(-p'^2) - I_1(-p'^2)]
\end{aligned} \tag{7.5}$$

where $p'^2 = (p^0)^2 - (p^1)^2$, $p''^2 = (p^2)^2 + (p^3)^2$, $f_{jik} f_{j'ik} = C\delta_{jj'}$ and the final integrals

$$I_r(k^2) = \int_0^1 dx \frac{x^r}{x(1-x)k^2 + \mu^2} \tag{7.6}$$

are finite. Hence the diagrams with $(LA_i)(LA_j)$ external terms are finite. All the other diagrams with two gauge field lines vanish or they are finite like the above. On the other hand the one-loop diagrams with two external gauge field lines and internal ghost lines vanish too because of the k -integration. Hence my conclusion is that there is no first order one-loop counterterms with two external gauge fields.

In order to see whether the gauge field coupling constant is renormalized one has to study the second order one-loop diagrams. All these diagrams with three external gauge fields and with two and three internal gauge fields have been written down. Their number is quite large, but they can be grouped using the above discrete symmetry. Investigating these diagrams, I found that they are all finite to. This implies that there is no first order coupling constant renormalization, which means that the first term of the function $\beta(\gamma)$ of the renormalization group equation vanishes.

8 DISCUSSION

For a long time, it was believed that there is no four dimensional renormalizable generally covariant Lagrangian model without higher order derivatives. Recall that this is one of the arguments which turned Physics research to strings. The present model shows that this belief is not true. The remarkable point is that the renormalizability is achieved through the standard technique of enhancing the symmetry. In the present model the Weyl (conformal) symmetry over the metric is extended to every vector of the integrable (null) tetrad $(\ell_\mu, n_\mu, m_\mu, \overline{m}_\mu)$. This symmetry was achieved after a slight modification of the Yang-Mills action. But this modification has the following severe consequences. The action is no longer metric dependent. Instead it is only complex structure dependent like the two dimensional string action. Besides this extended Weyl symmetry does not seem to permit the introduction of fermionic fields. All my efforts to find symmetric fermionic action terms have failed. This feature may not cause a problem to the physical content of the model, because some geometric solitons of the model have fermionic gyromagnetic ratio. This means that the present model may not be supersymmetrizable and may not even need a supersymmetrization to include fermions. The other essential difference between the present action and the ordinary Yang Mills action is at the generated static potential. The present action generates a confining linear static potential instead of the well known $\frac{1}{r}$ Coulomb potential of the ordinary Yang-Mills action. This means that the expected "quark confinement" is now perturbative without any reference to the not yet proved "infrared confinement" of ordinary gluonic action.

The present proof of the renormalizability of the model is based on the exclusion of all possible counterterms. It would be interesting to prove it using the conventional method of Ward identities, which are very complicated in the present gauge conditions. The first loop diagrams confirmed the renormalizability of the model and they indicate that it may be finite. In any case, it would be interesting to compute the loop diagrams with four external tetrads, which could generate the symmetric term (5.3). Finiteness of these diagrams would persuade us that the present model is something special.

It is well known that anomalies could destroy renormalizability. The finiteness of the first loop diagrams implies no first order anomalies, but they cannot be excluded to appear in higher orders.

In current terminology, a Lagrangian model is called finite if all its transition amplitudes on mass shell are finite without making use of any infinite renormalization either of the field or of the coupling constants. These amplitudes (on mass shell) do not depend on the regularization procedure or the imposed gauge condition, therefore their finiteness should not depend on these two choices either. The general Green functions of a finite field theoretical model may diverge, depending on the used gauge conditions. Apparently the existence of a gauge condition, which makes the Green functions finite, imply finiteness of the model. This formal reasoning works well in the case of supersymmetric Yang-Mills (SYM) model. It has been conjectured that the four dimensional $N = 4$ supersymmetric Yang-Mills (SYM) model is finite[5], while the six and

ten dimensional SYM models are not finite[6]. Therefore the fact that in the precise convenient gauge, which was used in the present calculations, the Green functions are finite, implies that the present model is also finite in the first order approximation. In a different gauge condition (e.g. Landau or Feynman) the Green functions may not be finite but the cross-sections must be finite.

References

- [1] Bogolioubov N. N. and Chirkov D. V. (1960), “Introduction à la Théorie Quantique des Champs”, Dunod, Paris.
- [2] Flaherty E. J. Jr (1974), Phys. Lett. A46, 391.
- [3] Flaherty E. J. Jr (1976), “Hermitian and Kählerian geometry in Relativity”, Lecture Notes in Physics 46, Springer, Berlin.
- [4] Leibbrandt G. (1975), Rev. Mod. Phys. 47, 849.
- [5] Mandelstam S. (1983), Nucl. Phys. B213, 149.
- [6] Ragiadakos C. N. (1983), Phys. Lett. B120, 142; Ragiadakos C. N. and Taylor J. G. (1983), Phys. Lett. B124, 201.
- [7] Ragiadakos C. N. (1988), “LEITE LOPES Festschrift - A pioneer physicist in the third world”, Edited by N. Fleury et al., World Scientific, Singapore.
- [8] Ragiadakos C. N. (1990), Phys. Lett. B251, 94.
- [9] Ragiadakos C. N. (1991), Phys. Lett. B269, 325..
- [10] Ragiadakos C. N. (1992), J. Math. Phys. 33, 122.
- [11] Ragiadakos C. N. (1999), Int. J. Math. Phys. A14, 2607.
- [12] 't Hooft G. and Veltman M. (1972), Nucl. Phys. B44, 189.